Algebraic Thinking and Classification
Session 1

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Algebraic Thinking	Creating and Identifying Patterns (Warm-Up)	3	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22		
	The Big Ideas of Algebra	4	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22	The Big Ideas of Algebraic Thinking, Algebraic Thinking: Making the Connection, Patterns, Functions, and Algebra SOL	
	Why is Algebraic Thinking Important?	9	K.17, K.18, 1.20, 1.21, 2.25, 2.26, 3.24, 3.25, 4.21, 4.22, 5.20, 5.21, 5.22	Patterns, Functions, and Algebra SOL, Why is Algebraic Thinking Important in the K-5 Curriculum?, Information Sheet on Patterns, Functions, and Algebra from NCTM, Algebraic Thinking in the NCTM Standards Summary Sheet	
Classification	What's In the Box? Tibby Warm-Up)	31	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20		32 piece set of attribute materials
	Play!	32	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, , 5.20		32 piece set of attribute materials
	Missing Pieces	33	K.17, 1.20, 2.25, 3.24, 4.21, 5.20		32 piece set of attribute materials
	20 Questions Game	34	4.21, 5.20		32 piece set of attribute materials
	Who Am I? Game	36	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 3.25, 4.21, 5.20,		32 piece set of attribute materials. Sample clue cards

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
	Differences - Train and Games	39	K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20,	Differences	32 piece set of attribute materials
Classification (continued)	Hidden Number Patterns	42	4.21, 5.20	Hidden Number Pattern Sheet	32 piece set of attribute materials
	Attribute Networks	45	4.21	Attribute Networks	32 piece set of attribute materials
	Two-Loop Problems	47	4.21, 5.20	Attribute Cards	32 piece set of attribute materials

Activity: Creating and Identifying Patterns (Warm-Up)

Format: Small Group

Objectives: Participants will develop a pattern in small groups and present it to the whole group. This activity is designed to allow participants to introduce themselves and for the whole group to determine the pattern shown by each small group.

Related SOL: Patterns, Functions, and Algebra Standards of Learning

Materials: None needed

Time Required: 15 minutes

Directions:

1. Divide participants into small groups of four.
2. Participants should introduce themselves in the small groups and discuss a possible pattern that exists in their group. Examples of possible patterns could be (brown hair, blonde hair, brown hair, blonde hair) or (glasses, no glasses, glasses, no glasses).
3. Ask each small group to stand up so that the pattern that they have selected is visible to the rest of the participants (e.g., brown hair, blonde hair, brown hair, blonde hair). The rest of the participants should make conjectures about what the pattern could be. The small group will then confirm and/or explain their pattern. They should then introduce themselves to the whole group (name, school division, grade level).

Activity: **The Big Ideas of Algebra**

Format: Small Group

Objectives: Participants will discuss the meaning of algebraic thinking and identify big mathematical ideas that constitute algebraic thinking in the K-5 curriculum.

Related SOL: Patterns, Functions, and Algebra Standards of Learning.

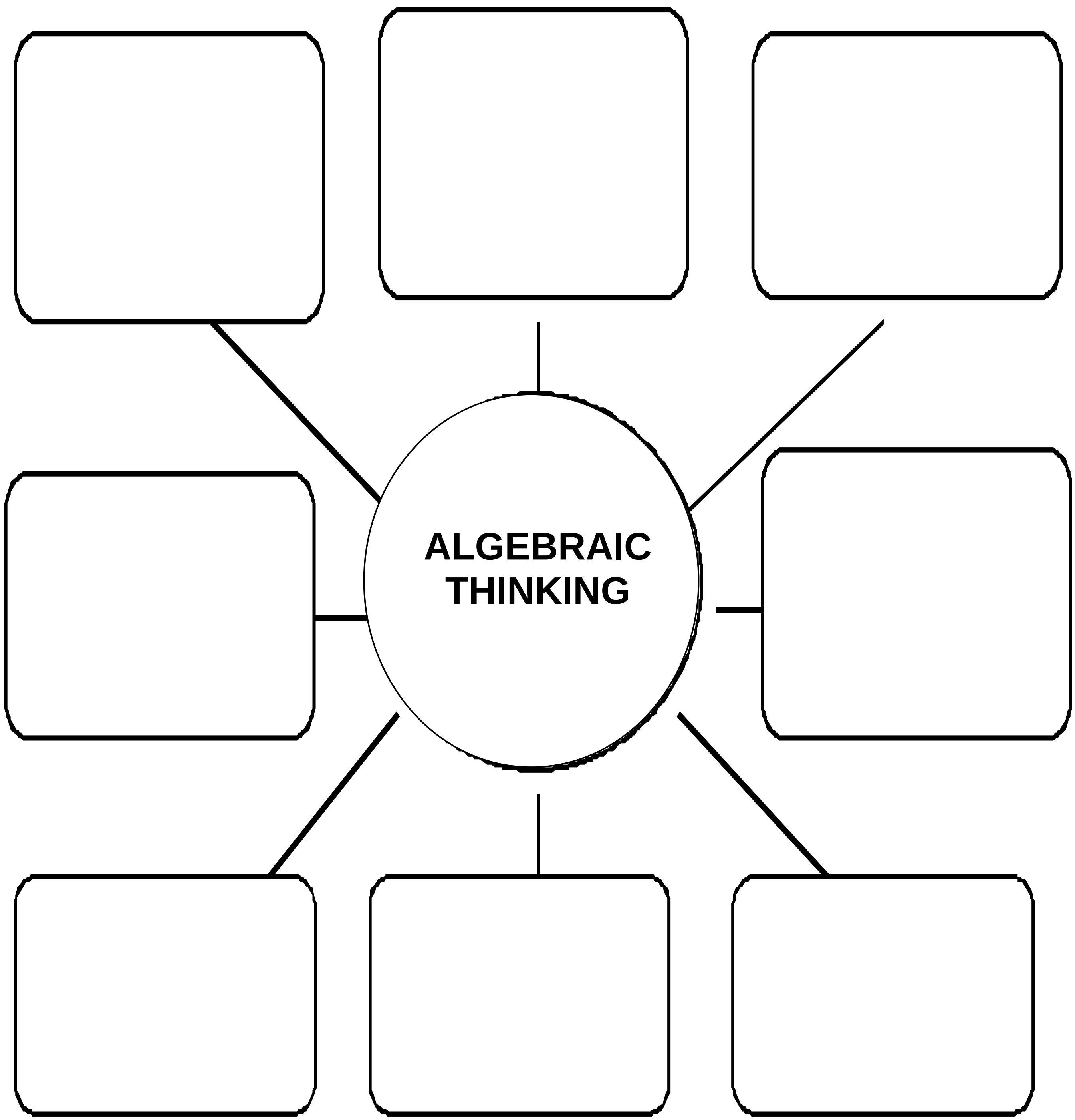
Materials: The Big Ideas of Algebraic Thinking and Algebraic Thinking: Making the Connections Activity Sheets; Standards of Learning Handout

Time Required: 15 minutes

Directions:

1. Divide participants into small groups of four (preferably with different grade levels represented).
2. Pose the question: "When you think of the term algebraic thinking, what do you think of? What are some of the key words in the Standards of Learning that constitute algebraic thinking?" Have participants refer to the Standards of Learning Sheet to look for key words and ideas. Assign a group recorder the task of writing the words the group associates with algebraic thinking on the handout. Give groups about 5 minutes to discuss.
3. Have groups contribute ideas. Record ideas in a webbing fashion to show how ideas connect on the transparency of "The Big Ideas of Algebraic Thinking". Discuss each briefly and give examples.
4. The big ideas should include such things as: patterns, functions, equations, variables, sorting and classifying, proportional reasoning, number relationships, expressions, and graphing.
5. Use the Algebraic Thinking: Making the Connections Activity Sheet to discuss how algebraic thinking is explicitly written in the SOL.

The Big Ideas of Algebraic Thinking



Virginia Standards of Learning

Patterns, Functions and Algebra Strand K-5

K.17 The student will sort and classify objects according to similar attributes (size, shape, and color).

K.18 The student will identify, describe, and extend a repeating relationship (pattern) found in common objects, sounds, and movements.

1.20 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

1.21 The student will recognize, describe, extend, and create a wide variety of patterns, including rhythmic, color, shape, and numeric. Patterns will include both growing and repeating patterns. Concrete materials and calculators will be used by students.

2.25 The student will identify, create, and extend a wide variety of patterns, using numbers, concrete objects, and pictures.

2.26 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. Examples include: 3 + __ = 7, or 9 - __ = 2. Students will create story problems using the numerical sentences.

3.24 The student will recognize and describe a variety of patterns formed using concrete objects, numbers, tables, and pictures, and extend the pattern, using the same or different forms (concrete objects, numbers, tables, and pictures).

3.25 The student will a) investigate and create patterns involving numbers, operations (addition and multiplication), and relations that model the identity and commutative properties for addition and multiplication; and b) demonstrate an understanding of equality by recognizing that the equal sign (=) links equivalent quantities, such as $4 \cdot 3 = 2 \cdot 6$.

4.21 The student will recognize, create, and extend numerical and geometric patterns, using concrete materials, number lines, symbols, tables, and words.

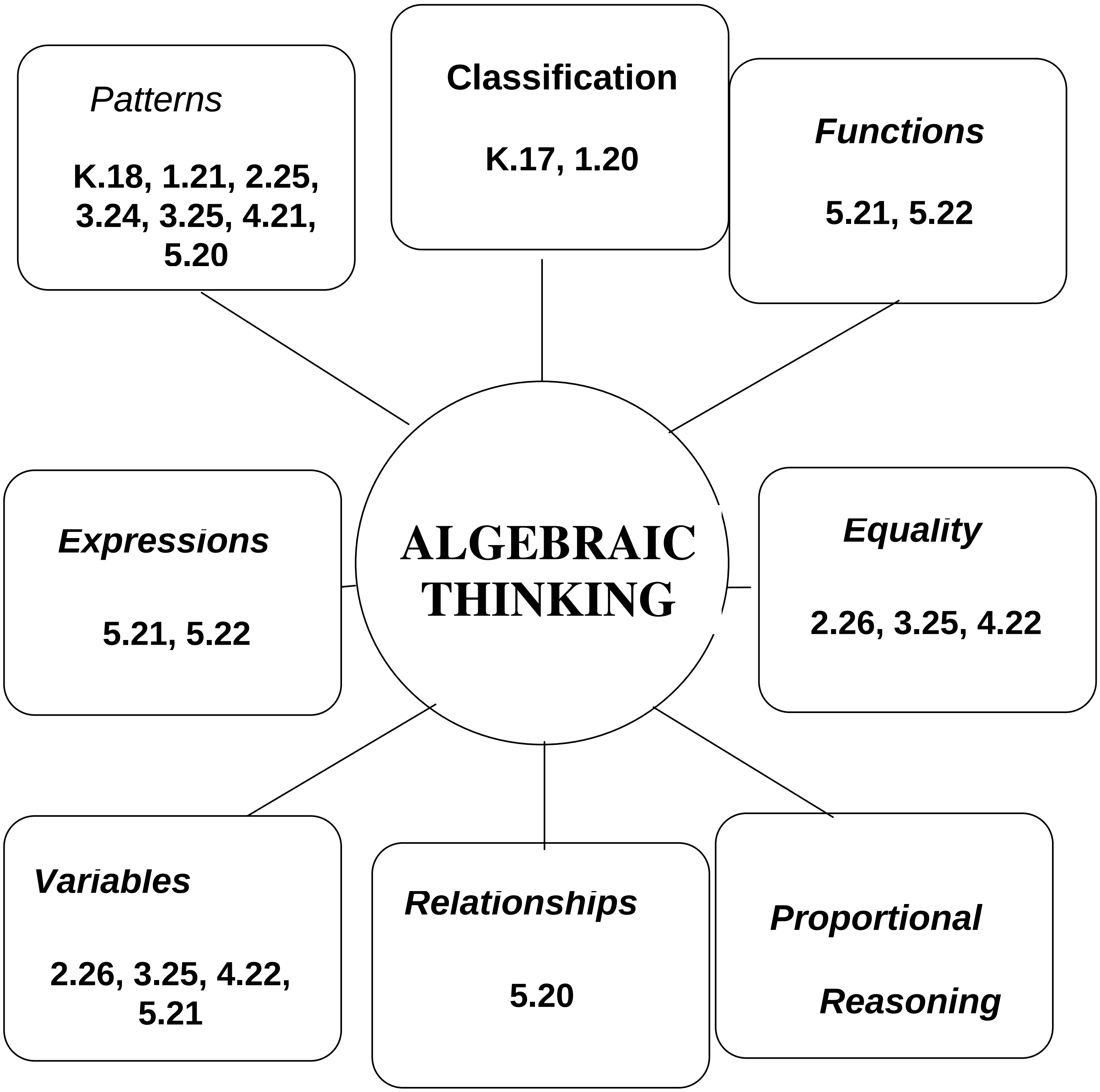
4.22 The student will recognize and demonstrate the meaning of equality, using symbols representing numbers, operations, and relations [e.g., 3 + 5 = 5 + 3 and 15 + (35 + 16) = (15 + 35) + 16].

5.20 The student will analyze the structure of numerical and geometric patterns (how they change or grow) and express the relationship, using words, tables, graphs, or a mathematical sentence. Concrete materials and calculators will be used.

5.21 The student will
- a) investigate and describe the concept of variable;
- b) use a variable expression to represent a given verbal quantitative expression, involving one operation; and
- c) write an open sentence to represent a given mathematical relationship, using a variable.

5.22 The student will create a problem situation based on a given open sentence using a single variable.

Algebraic Thinking: Making the Connections



Activity: **Why is Algebraic Thinking Important?**

Format: Whole group, Mini-Lecture

Objectives: Participants will understand the rationale for including algebraic topics in the K-5 curriculum by discussing how the big ideas of algebraic thinking are connected with the NCTM *Principles and Standards for School Mathematics.*

Related SOL: Patterns, Functions, and Algebra Standards of Learning

Materials: Why is Algebraic Thinking Important in the K-5 Curriculum? Activity Sheet, Algebraic Thinking in the NCTM *Principles and Standards for School Mathematics* Activity Sheet, copies for each participant of NCTM resource for discussion, "Why is Algebraic Thinking Important in the K-5 Curriculum?"

Time Required: 15 minutes

Directions:

1. Distribute copies of the NCTM resource for discussion, "Why is Algebraic Thinking Important in the K-5 Curriculum? ". Ask participants to review the information. Use the transparency of "Why is Algebraic Thinking Important in the K-5 Curriculum?" to guide the discussion. Discuss each of the bullets on the transparency.
2. Discuss the National Council of Teachers of Mathematics organization and the 2000 Principles and Standards for School Mathematics document. Discuss how algebraic thinking is emphasized in the Standards and how the Virginia SOL reflect the recommendations made in the document. Use the transparencies of "Algebraic Thinking in the NCTM Principles and Standards for School Mathematics".

Resource for Discussion from the NCTM *Principles and Standards for School Mathematics* on "Why is Algebraic Thinking Important in the K-5 Curriculum?"

NCTM Standard 2: Patterns, Functions, and Algebra
Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Elaboration: Grades Pre-K-2
Algebraic concepts can evolve and continue to develop during prekindergarten through grade 2. They will be manifested through work with classification, patterns and relations, operations with whole numbers, explorations of function, and step-by-step processes. Although the concepts discussed in this Standard are algebraic, this does not mean that students in the early grades are going to deal with the symbolism often taught in a traditional high school algebra course.

Even before formal schooling, children develop beginning concepts related to patterns, functions, and algebra. They learn repetitive songs, rhythmic chants, and predictive poems that are based on repeating and growing patterns. The recognition, comparison, and analysis of patterns are important components of a student's intellectual development. When students notice that operations seem to have particular properties, they are beginning to think algebraically. For example, they realize that changing the order in which two numbers are added does not change the result or that adding zero to a number leaves that number unchanged. Students' observations and discussions of how quantities relate to one another lead to initial experiences with function relationships, and their representations of mathematical situations using concrete objects, pictures, and symbols are the beginnings of mathematical modeling. Many of the step-by-step processes that students use form the basis of understanding iteration and recursion.

Standard 2 Focus Areas for Grades Pre-K-2

Understand patterns, relations, and functions
In grades preK-2, all students should:

- sort, classify, and order objects by size, number, and other properties;
- recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;

- analyze how both repeating and growing patterns are generated.

Represent and analyze mathematical situations and structures using algebraic symbols

In grades preK-2, all students should

- illustrate general principles and properties of operations, such as commutativity, using specific numbers;
- use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.

Use mathematical models to represent and understand quantitative relationships

In grades preK-2, all students should

- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols

Analyze change in various contexts

In grades preK-2, all students should

- describe qualitative change, such as a student's growing taller;
- describe quantitative change, such as a student's growing two inches in one year.

Discussion

Algebraic concepts can evolve and continue to develop during prekindergarten through grade 2. They will be manifested through work with classification, patterns and relations, operations with whole numbers, explorations of function, and step-by-step processes. Although the concepts discussed in this Standard are algebraic, this does not mean that students in the early grades are going to deal with the symbolism often taught in a traditional high school algebra course.

Even before formal schooling, children develop beginning concepts related to patterns, functions, and algebra. They learn repetitive songs, rhythmic chants, and predictive poems that are based on repeating and growing patterns. The recognition, comparison, and analysis of patterns are important components of a student's intellectual development. When students notice that operations seem to have particular properties, they are beginning to think algebraically. For example, they realize that changing the order in which two numbers are added does not change the result or that adding zero to a number leaves that number unchanged. Students' observations and discussions of how quantities relate to one another lead to initial experiences with function relationships, and their representations of mathematical situations using concrete objects, pictures, and symbols are the beginnings of mathematical modeling. Many of the step-by-step processes that students use form the basis of understanding iteration and recursion.

Understand patterns, relations, and functions

Sorting, classifying, and ordering facilitate work with patterns, geometric shapes, and data. Given a package of assorted stickers, children quickly notice many differences among the items. They can sort the stickers into groups having similar traits such as color, size, or design and order them from smallest to largest. Caregivers and teachers should elicit from children the criteria they are using as they sort and group objects. Patterns are a way for young students to recognize order and to organize their world and are important in all aspects of mathematics at this level. Preschoolers recognize patterns in their environment and, through experiences in school, should become more skilled in noticing patterns in arrangements of objects, shapes, and numbers and in using patterns to predict what comes next in an arrangement. Students know, for example, that "first comes breakfast, then school," and "Monday we go to art, Tuesday we go to music." Students who see the digits "0, 1, 2, 3, 4, 5, 6, 7, 8, 9" repeated over and over will see a pattern that helps them learn to count to 100—a formidable task for students who do not recognize the pattern.

Teachers should help students develop the ability to form generalizations by asking such questions as "How could you describe this pattern?" or "How can it be repeated or extended?" or "How are these patterns alike?" For example, students should recognize that the color pattern "blue, blue, red, blue, blue, red" is the same in form as "clap, clap, step, clap, clap, step." This recognition lays the foundation for the idea that two very different situations can have the same mathematical » features and thus are the same in some important ways. Knowing that each pattern above could be described as having the form AABAAB is for students an early introduction to the power of algebra.

By encouraging students to explore and model relationships using language and notation that is meaningful for them, teachers can help students see different relationships and make conjectures and generalizations from their experiences with numbers. Teachers can, for instance, deepen students' understanding of numbers by asking them to model the same quantity in many ways—for example, eighteen is nine groups of two, 1 ten and 8 ones, three groups of six, or six groups of three. Pairing counting numbers with a repeating pattern of objects can create a function (see fig. 4.7) that teachers can explore with students: What is the second shape? To continue the pattern, what shape comes next? What number comes next when you are counting? What do you notice about the numbers that are beneath the triangles? What shape would 14 be?

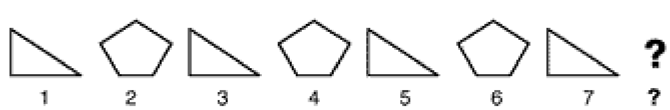


Fig. **4.7.** Pairing counting numbers with a repeating pattern

Students should learn to solve problems by identifying specific processes. For example, when students are skip-counting three, six, nine, twelve, ..., one way to obtain the next term is to add three to the previous number. Students can use a similar process to compute how much to pay for seven balloons if one balloon costs 20¢. If they recognize the sequence 20, 40, 60, ... and continue to add 20, they can find the cost for seven balloons. Alternatively, students can realize that the total amount to be paid is determined by the number of balloons bought and find a way to compute the total directly. Teachers in grades 1 and 2 should provide experiences for students to learn to use charts and tables for recording and organizing information in varying formats (see figs. 4.8 and 4.9). They also should discuss the different notations for showing amounts of money. (One balloon costs 20¢, or $0.20, and seven balloons cost $1.40.)

Cost of Balloons

Number of Balloons	Cost of Balloons in Cents
1	20
2	40
3	60
4	80
5	?
6	?
7	?

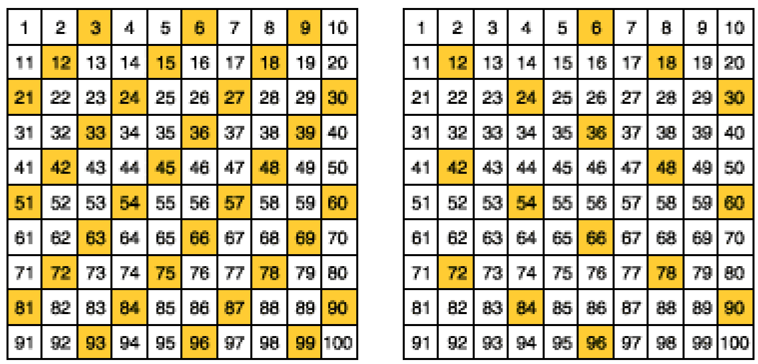
Fig. **4.8.** A vertical chart for recording and organizing information

Cost of Balloons

Number of balloons	1	2	3	4	5	6	7
Cost of balloons in cents	20	40	60	80	?	?	?

Fig. **4.9.** A horizontal chart for recording and organizing information

Skip-counting by different numbers can create a variety of patterns on a hundred chart that students can easily recognize and describe (see fig. 4.10). Teachers can simultaneously use hundred charts to help students learn about number patterns and to assess students' understanding of counting patterns. By asking questions such as "If you count by tens beginning at 36, what number would you color next?" and "If you continued counting by tens, would you color 87?" teachers can observe whether students understand the correspondence between the visual pattern formed by the shaded numbers and the counting pattern. Using a calculator and a hundred chart enables the students to see the same pattern in two different formats.



Counting by threes   Counting by sixes

Fig. **4.10.** Skip-counting on a hundred chart

Represent and analyze mathematical situations and structures using algebraic symbols

Two central themes of algebraic thinking are appropriate for young students. The first involves making generalizations and using symbols to represent mathematical ideas, and the second is representing and solving problems (Carpenter and Levi 1999). For example, adding pairs of numbers in different orders such as 3 + 5 and 5 + 3 can lead students to infer that when two numbers are added, the order does not matter. As students generalize from observations about number and operations, they are forming the basis of algebraic thinking.

Similarly, when students decompose numbers in order to compute, they often use the associative property for the computation. For instance, they may compute 8 + 5, saying, "8 + 2 is 10, and 3 more is 13." Students often discover and make generalizations about other properties. Although it is not necessary to introduce vocabulary such as *commutativity* or *associativity,* teachers must be aware of the algebraic properties used by students at this age. They should build students' understanding of the importance of their observations about mathematical situations and challenge them to investigate whether specific observations and conjectures hold for all cases.

Teachers should take advantage of their observations of students, as illustrated in this story drawn from an experience in a kindergarten class.

The teacher had prepared two groups of cards for her students. In the first group, the number on the front and back of each card differed by 1. In the second group, these numbers differed by 2.

The teacher showed the students a card with 12 written on it and explained, "On the back of this card, I've written another number." She turned the card over to show the number 13. Then she showed the students a second card with 15 on the front and 16 on the back. » As she continued showing the students the cards, each time she asked the students, "What do you think will be on the back?" Soon the students figured out that she was adding 1 to the number on the front to get the number on the back of the card.

Then the teacher brought out a second set of cards. These were also numbered front and back, but the numbers differed by 2, for example, 33 and 35, 46 and 48, 22 and 24. Again, the teacher showed the students a sample card and continued with other cards, encouraging them to predict what number was on the back of each card. Soon the students figured out that the numbers on the backs of the cards were 2 more than the numbers on the fronts.

When the set of cards was exhausted, the students wanted to play again. "But," said the teacher, "we can't do that until I make another set of cards." One student spoke up, "You don't have to do that, we can just flip the cards over. The cards will all be minus 2."

As a follow-up to the discussion, this teacher could have described what was on each group of cards in a more algebraic manner. The numbers on the backs of the cards in the first group could be named as "front number plus 1" and the second as "front number plus 2." Following the student's suggestion, if the cards in the second group were flipped over, the numbers on the backs could then be described as "front number minus 2." Such activities, together with the discussions and analysis that follow them, build a foundation for understanding the inverse relationship.

Through classroom discussions of different representations during the pre-K–2 years, students should develop an increased ability to use symbols as a means of recording their thinking. In the earliest years, teachers may provide scaffolding for students by writing for them until they have the ability to record their ideas. Original representations remain important throughout the students' mathematical study and should be encouraged. Symbolic representation and manipulation should be embedded in instructional experiences as another vehicle for understanding and making sense of mathematics.

Equality is an important algebraic concept that students must encounter and begin to understand in the lower grades. A common explanation of the equals sign given by students is that "the answer is coming," but they need to recognize that the equals sign indicates a relationship—that the quantities on each side are equivalent, for example, 10 = 4 + 6 or 4 + 6 = 5 + 5. In the later years of this grade band, teachers should provide opportunities for students to make connections from symbolic notation to the representation of the equation. For example, if a student records the addition of four 7s as shown on the left in figure 4.11, the teacher could show a series of additions correctly, as shown on the right, and use a balance and cubes to demonstrate the equalities.

$7+7=14+7=21+7=28$

$$\begin{aligned} 7+7&=14 \\ 14+7&=21 \\ 21+7&=28 \end{aligned}$$

Fig. **4.11.** A student's representation of adding four 7s (left) and a teacher's correct representation of the same addition

Use mathematical models to represent and understand quantitative relationships

Students should learn to make models to represent and solve problems. For example, a teacher may pose the following problem:

> There are six chairs and stools. The chairs have four legs and the stools have three legs. All together, there are 20 legs. How many chairs and how many stools are there?

One student may represent the situation by drawing six circles and then putting tallies inside to represent the number of legs. Another student may represent the situation by using symbols, making a first guess that the number of stools and chairs is the same and adding 3 + 3 + 3 + 4 + 4 + 4. Realizing that the sum is too large, the student might adjust the number of chairs and stools so that the sum of their legs is 20.

Analyze change in various contexts

Change is an important idea that students encounter early on. When students measure something over time, they can describe change both qualitatively (e.g., "Today is colder than yesterday") and quantitatively (e.g., "I am two inches taller than I was a year ago"). Some changes are predictable. For instance, students grow taller, not shorter, as they get older. The understanding that most things change over time, that many such changes can be described mathematically, and that many changes are predictable helps lay a foundation for applying mathematics to other fields and for understanding the world.

Resource for Discussion from the NCTM *Principles and Standards for School Mathematics* on "Why is Algebraic Thinking Important in the K-5 Curriculum?"

NCTM Standard 2: Patterns, Functions, and Algebra
Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Elaboration: Grades 3-5
Although *algebra* is a word that has not commonly been heard in grades 3–5 classrooms, the mathematical investigations and conversations of students in these grades frequently include elements of algebraic reasoning. These experiences and conversations provide rich contexts for advancing mathematical understanding and are also an important precursor to the more formalized study of algebra in the middle and secondary grades. In grades 3–5, algebraic ideas should emerge and be investigated as students—

- identify or build numerical and geometric patterns;
- describe patterns verbally and represent them with tables or symbols;
- look for and apply relationships between varying quantities to make predictions;
- make and explain generalizations that seem to always work in particular situations;
- use graphs to describe patterns and make predictions;
- explore number properties;
- use invented notation, standard symbols, and variables to express a pattern, generalization, or situation.

Focus Areas for Grades 3-5

Understand patterns, relations, and functions
In grades 3-5, all students should

- describe, extend, and make generalizations about geometric and numeric patterns;
- represent and analyze patterns and functions, using words, tables, and graph

Represent and analyze mathematical situations and structures using algebraic symbols

In grades 3-5, all students should

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
- identify such properties as commutativity, associativity, and distributivity, and use them to compute with whole numbers;
- represent the idea of a variable as an unknown quantity using a letter or a symbol;
- express mathematical relationships using equations.

Use mathematical models to represent and understand quantitative relationships

In grades 3-5, all students should-

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Analyze change in various contexts

In grades 3-5, all students should-

- investigate how a change in one variable relates to a change in a second variable;
- identify and describe situations with constant or varying rates of change and compare them.

Discussion

Understand patterns, relations, and functions

In grades 3–5, students should investigate numerical and geometric patterns and express them mathematically in words or symbols. They should analyze the structure of the pattern and how it grows or changes, organize this information systematically, and use their analysis to develop generalizations about the mathematical relationships in the pattern. For example, a teacher might ask students to describe patterns they see in the "growing squares" display (see fig. 5.3) and express the patterns in mathematical sentences. Students should be encouraged to explain these patterns verbally and to make predictions about what will happen if the sequence is continued.

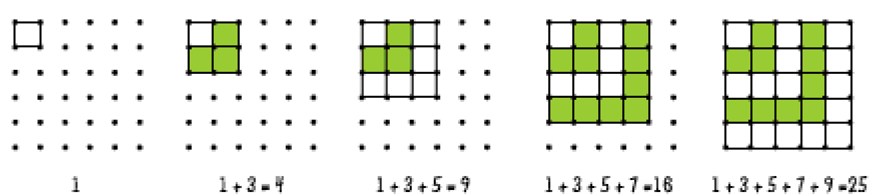


Fig. **5.3.** Expressing "growing squares" in mathematical sentences (Adapted from Burton et al. 1992, p. 6)

In this example, one student might notice that the area changes in a predictable way—it increases by the next odd number with each new square. Another student might notice that the previous square always fits into the "corner" of the next-larger square. This observation might lead to a description of the area of a square as equal to the area of the previous square plus "its two sides and one more." A student might represent his thinking as in figure 5.4.»

Area of a 5x5 square = area of a 4x4 square +4+4+1

Fig. **5.4.** A possible student observation about the area of the 5 × 5 square in the "growing squares" pattern

Examples like this one give the teacher important opportunities to engage students in thinking about how to articulate and express a generalization—"How can we talk about how this pattern works for a square of any size?" Students in grade 3 should be able to predict the next element in a sequence by examining a specific set of examples. By the end of fifth grade, students should be able to make generalizations by reasoning about the structure of the pattern. For example, a fifth-grade student might explain that "if you add the first n odd numbers, the sum is the same as n × n."

As they study ways to measure geometric objects, students will have opportunities to make generalizations based on patterns. For example, consider the problem in figure 5.5. Fourth graders might make a table (see fig. 5.6) and note the iterative nature of the pattern. That is, there is a consistent relationship between the surface area of one tower and the next-bigger tower: "You add four to the previous number." Fifth graders could be challenged to justify a general rule with reference to the geometric model, for example, "The surface area is always four times the number of cubes plus two more because there are always four square units around each cube and one extra on each end of the tower." Once a relationship is established, students should be able to use it to answer questions like, "What is the surface area of a tower with fifty cubes?" or "How many cubes would there be in a tower with a surface area of 242 square units?"

What is the surface area of each tower of cubes (include the bottom)?
As the towers get taller, how does the surface area change?

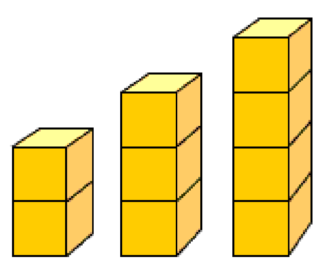


Fig. **5.5.** Finding surface areas of towers of cubes

Number of Cubes (N)	Surface Area in Square Units (S)
1	6
2	10
3	14
4	18

Fig. 5.6**.** A table used in the "tower of cubes" problem

Represent and analyze mathematical situations and structures using algebraic symbols
In grades 3–5, students can investigate properties such as commutativity, associativity, and distributivity of multiplication over addition. Is 3×5 the same as 5×3? Is 15×27 equal to 27×15? Will reversing the factors always result in the same product? What if one of the factors is a decimal number (e.g., 1.5×6)? An area model can help students see that two factors in either order have equal products, as represented by congruent rectangles with different orientations (see fig. 5.7). »

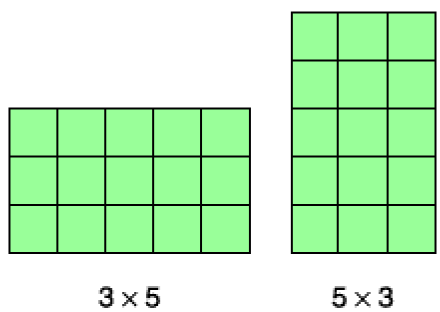
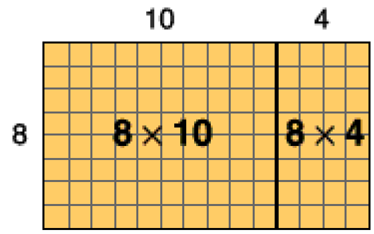


Fig. **5.7.** Area models illustrating the commutative property of multiplication

An area model can also be used to investigate the distributive property. For example, the representation in figure 5.8 shows how 8×14 can be decomposed into 8×10 and 8×4.



$80 + 32 = 112$

Fig. **5.8.** Area model showing the distributive property of multiplication

As students learn about the meaning of multiplication and develop strategies to solve multiplication problems, they will begin to use properties such as distributivity naturally (Schifter 1999). However, discussion about the properties themselves, as well as how they serve as tools for solving a range of problems, is important if students are to add strength to their intuitive notions and advance their understanding of multiplicative structures. For example, students might explore questions such as these: Why can't 24×32 be solved by adding the results of 20×30 and 4×2? If a number is tripled, then tripled again, what is the relationship of the result to the original number? Analyzing the properties of operations gives students opportunities to extend their thinking and to build a foundation for applying these understandings to other situations.

At this grade band the idea and usefulness of a variable (represented by a box, letter, or symbol) should also be emerging and developing more fully. As students explore patterns and note relationships, they should be encouraged to represent their thinking. In the example showing the sequence of squares that grow (fig. 5.3), students are beginning to use the idea of a variable as they think about how to describe a rule for finding the area of any square from the pattern they have observed. As students become more experienced in investigating, articulating, and justifying generalizations, they can begin to use variable notation and equations to represent their thinking. Teachers will need to model how to represent thinking in the form of equations. In this way, they can » help students connect the ways they are describing their findings to mathematical notation. For example, a student's description of the surface area of a cube tower of any size ("You get the surface area by multiplying the number of cubes by 4 and adding 2") can be recorded by the teacher as $S = 4 \times n + 2$. Students should also understand the use of a variable as a placeholder in an expression or equation. For example, they should explore the role of n in the equation $80 \times 15 = 40 \times n$ and be able to find the value of n that makes the equation true.

Use mathematical models to represent and understand quantitative relationships
Historically, much of the mathematics used today was developed to model real-world situations, with the goal of making predictions about those situations. As patterns are identified, they can be expressed numerically, graphically, or symbolically and used to

predict how the pattern will continue. Students in grades 3–5 develop the idea that a mathematical model has both descriptive and predictive power.

Students in these grades can model a variety of situations, including geometric patterns, real-world situations, and scientific experiments. Sometimes they will use their model to predict the next element in a pattern, as students did when they described the area of a square in terms of the previous smaller square (see fig. 5.3). At other times, students will be able to make a general statement about how one variable is related to another variable: If a sandwich costs $3, you can figure out how many dollars any number of sandwiches costs by multiplying that number by 3 (two sandwiches cost $6, three sandwiches cost $9, and so forth). In this case, students have developed a model of a proportional relationship: the value of one variable (total cost, C) is always three times the value of the other (number of sandwiches, S), or $C = 3 \bullet S$.

In modeling situations that involve real-world data, students need to know that their predictions will not always match observed outcomes for a variety of reasons. For example, data often contain measurement error, experiments are influenced by many factors that cause fluctuations, and some models may hold only for a certain range of values. However, predictions based on good models should be reasonably close to what actually happens.

Students in grades 3–5 should begin to understand that different models for the same situation can give the same results. For example, as a group of students investigates the relationship between the number of cubes in a tower and its surface area, several models emerge. One student thinks about each side of the tower as having the same number of units of surface area as the number of cubes (n). There are four sides and an extra unit on each end of the tower, so the surface area is four times the number of cubes plus two ($4 \bullet n + 2$). Another student thinks about how much surface area is contributed by *each* cube in the tower: each end cube contributes five units of surface area and each "middle" cube contributes four units of surface area. Algebraically, the surface area would be $2 \bullet 5 + (n - 2) \bullet 4$. For a tower of twelve cubes, the first student thinks, "4 times 12, that's 48, plus 2 is 50." The second student thinks, "The two end cubes each have 5, so that's 10. There are 10 » more cubes. They each have 4, so that's 40. 40 plus 10 is 50." Students in this grade band may not be able to show how these solutions are algebraically equivalent, but they can recognize that these different models lead to the same solution.

Analyze change in various contexts

Change is an important mathematical idea that can be studied using the tools of algebra. For example, as part of a science project, students might plant seeds and record the growth of a plant. Using the data represented in the table and graph (fig. 5.9), students can describe how the rate of growth varies over time. For example, a student might express the rate of growth in this way: "My plant didn't grow for the first four days, then it grew slowly for the next two days, then it started to grow faster, then it slowed down again." In this situation, students are focusing not simply on the height of the plant each day but on what has happened between the recorded heights. This work is a precursor to later, more focused attention on what the slope of a line represents, that is, what the steepness of the line shows about the rate of change. Students should have opportunities to study situations that display different patterns of change—change that occurs at a constant rate, such as someone walking at a constant speed, and rates of change that increase or decrease, as in the growing-plant example.

Time (days)	Height (cm)	Change (cm)
0	0	
2	0	0
4	0	0
6	1	1
8	2	1
10	4	2
12	6	2
14	7.5	1.5
16	8.5	1
18	8.5	0
20	9	0.5

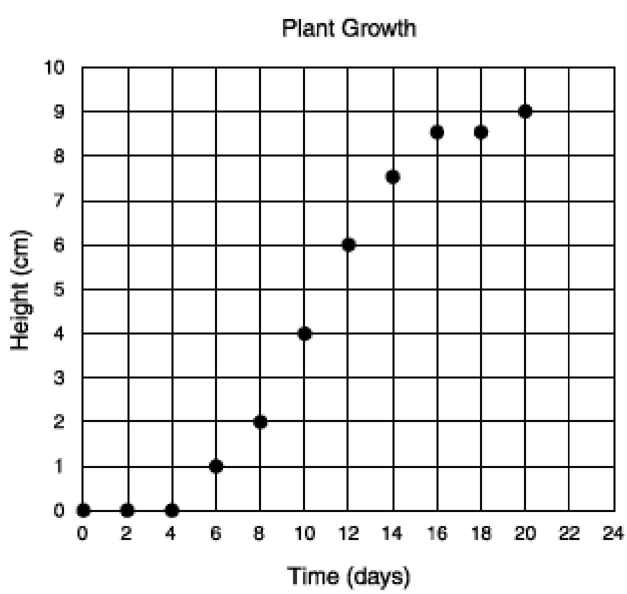


Fig. **5.9.** A table and graph showing growth of a plant

Reference: *Principles and Standards for School Mathematics* Electronic, 2000, National Council of Teachers of Mathematics

Why is Algebraic Thinking Important in the K-5 Curriculum?

1. **Algebraic thinking** provides the foundation for students to move beyond the specific computational skills to the general manipulation of symbols.
   Examples:
   $3 + \square = 7$ $\quad\quad$ $3 + x = 7$

   $2(3 + 5) = 2(3) + 2(5)$ $\quad\quad$ $2(a + b) = 2(a) + 2(b)$

2. **Algebraic thinking** is looking for, expecting, and understanding patterns.
   a. Repeating and growing patterns can be found in numbers.
   Find and continue the pattern:
   1, 2, 1, 2, ___, ___, ___
   1, 3, 5, 9, ___, ___, ___
   1, 3, 6, 10, 15, 21, ___, ___, ___

   b. Patterns are a vehicle that enables children to make sense of the world around them.
   Examples:
   Rhythmic patterns, seasons, phases of the moon

   c. Using patterns to solve problems is an extremely powerful tool for making connections in mathematics.
   Examples:
   Doubles: $2 + 2 = 4$
   Doubles Plus One: $2 + 3 = 5$
   Skip Counting 3, 6, 9, 12, 15, 18, ...
   Function Machines

$\square$	Δ
3	4
4	5
5	6

   function: $\square + 1 = \Delta$

3. **Algebraic thinking** is fundamental to functioning in business, industry, science, technology and daily life.
   Daily Life Example:
   Determining the cost for a McDonald's fast food order
   3 Cheeseburger + 2 fries + 2 soda+ 1 shake= ?
   $3\ C + 2F + 2\ S + 1\ K = 3\ (.99) + 2\ (.89) + 2\ (.79) + 1\ (.89) = ?$
   Second Example:
   Best Buys 5 for \$3.00 or 3 for \$2.00

Technology Example:
Computer Spreadsheets - Models use algebraic reasoning and symbolic notation

4. **Algebraic thinking** is the gatekeeper for higher mathematics and science courses.
   Example:
   - Algebra is a prerequisite to other high school mathematics courses
   - Success in Algebra is tied to gradation requirements
   - The foundation for algebraic thinking must begin in Kindergarten.

5. **Algebraic thinking** is a critical filter for employment and advanced training.
   Example:
   - More than 75% of all jobs require proficiency in fundamental algebraic concepts.

Algebraic Thinking in the NCTM *Standards*

In grades K-4, the mathematics curriculum should include the study of patterns and relationships so that students can:

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can:

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems.

In grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can:

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems.

Overview of Classification

Key Idea: Classification Using Attribute Block Materials

Description: Participants will explore the concept of classifying which is a basic process of mathematical thinking that is essential to many concepts that are developed in the grades K - 5 mathematics curriculum. Classification involves the understanding of relationships. Classification activities, which require observing likenesses and differences can be presented through problem solving situations and provide students with the opportunity to develop logical reasoning abilities. Logical reasoning skills and especially the meaningful use of the language of logic (e.g., if-then, and, or, not, all, some, etc.) are valuable across all areas of mathematics. An understanding of classification, or the recognition of the various attributes of items, is also an essential skill to patterning (e.g., extending, exploring, and creating patterns or sequences). These classification skills can be taught through a variety of materials (e.g., collections of leaves, buttons, etc), however, attribute pieces will be the manipulative used for this session.

Attribute Materials: *Attribute materials* are sets of objects that lend themselves to being sorted and classified in different ways. Natural or *unstructured* attribute materials include such things as seashells, leaves, the children themselves, or the set of the children's shoes. The *attributes* refer to the characteristic, quality or trait of the item and are the means by which materials can be sorted. For example, hair color, height, and gender are attributes of children. Each attribute has a number of different *values*: for example, blond, brown or red (for the attribute of hair color), tall or short (for height), male or female (for gender).

A *structured* set of attribute pieces has exactly one piece for every possible combination of values for each attribute. For example, several commercial sets of plastic attribute materials have four attributes: color (red, yellow, blue), shape (circle, triangle, rectangle, square, hexagon), size (big, little), and thickness (thick, thin). In the set just described there is exactly one large, red, thin, triangle as well as one each of all other combinations. The specific values, number of values, or number of attributes that a set may have is not important.

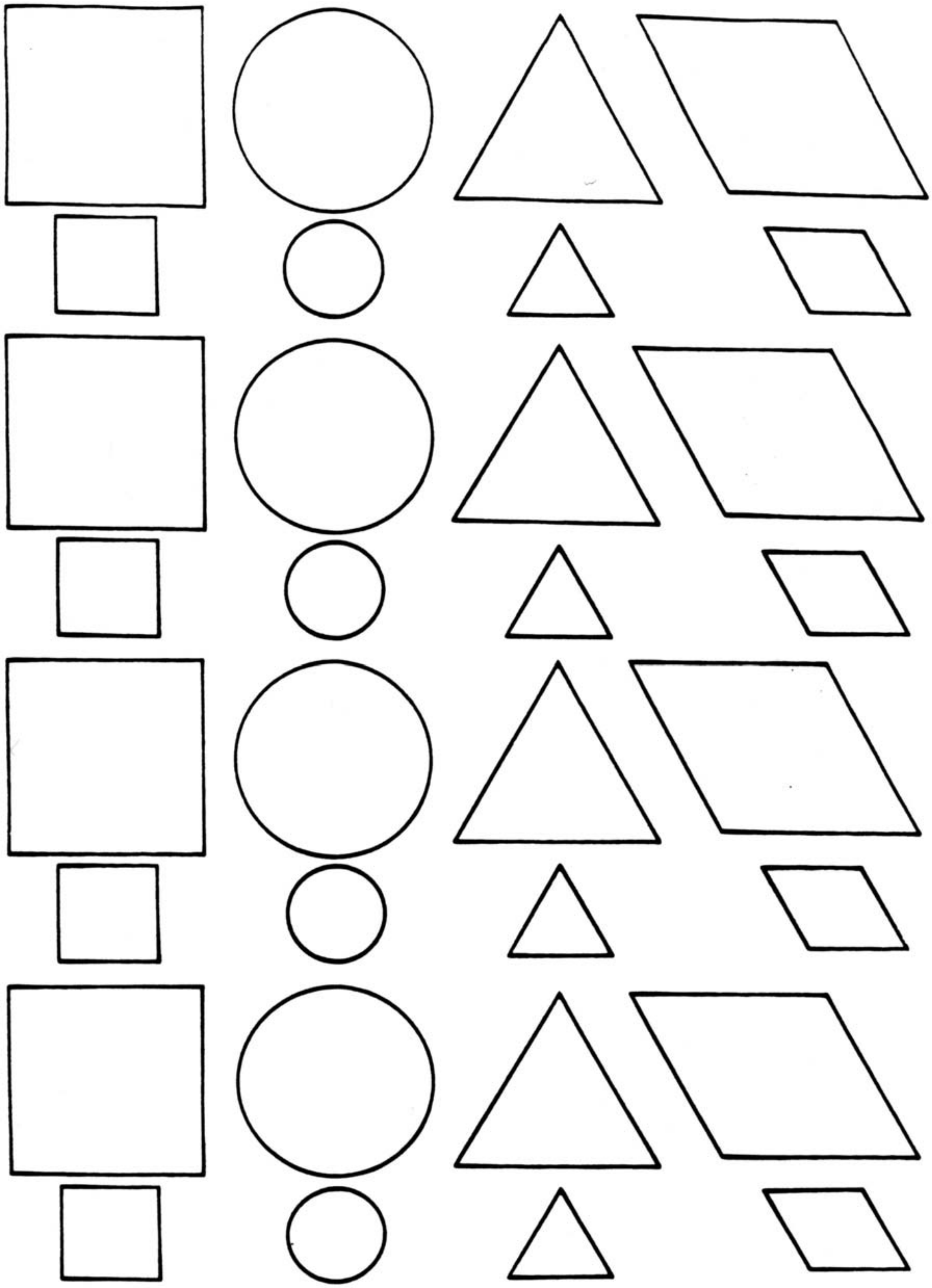
The value of using structured attribute materials (instead of unstructured materials) is that the attributes and values are very clearly identified and easily articulated to students. There is no confusion or argument concerning what values a particular piece possesses. In this way we can focus our attention in the activities on the reasoning skills that the materials and activities are meant to serve. Even though a nice set of attribute pieces may contain geometric shapes or different colors and sizes, they are not very good materials for teaching shape, color, or size. A set of attribute shapes does not provide enough variability in any of the shapes to help students develop anything but very limited geometric ideas. In fact, simple shapes, primary colors, and two sizes are usually chosen because they are most easily discriminated and identified by even the youngest of students.

Most attribute activities are best done in a teacher-directed format. Young children can sit on the floor in a large circle where all can see and have access to the materials. Older children can work in groups of four to six students, each group with its own set of materials. Older children can work with sets of attribute materials that have a greater number of pieces and attributes (i.e., the 60 piece set rather than the 32 piece set). In that format, problems can be addressed to the full class and groups can explore them independently. All activities should be conducted in an easygoing manner that encourages risks, good thinking, attentiveness and discussion of ideas. The atmosphere should be non-threatening, non-punitive, and non-evaluative.

You may wish to motivate primary students to think about attributes by reading them the book titled The Important Book by Margaret Wise Brown. As an extension, you may wish to have students develop their own "important book" about themselves as a project during your work with classification.

Constructing A Set of Attribute Blocks

To make four sets of attribute blocks, print the "Attribute Block" pattern (found on the next page) onto four different colors of heavy paper (red, yellow, blue, green). You may wish to laminate the pieces before you cut them out. Using this pattern, you will create four sets with 32 pieces: Four Colors (red, yellow, blue, green); Four Shapes (circle, triangle, rhombus [diamond], square); and Two Sizes (big, little). Make enough sets for your classroom so that two to four students share one set.



Attribute Block Pattern Warm-Up Activity: Tibby

Select an attribute, such as the color of a participant's shirt, hair, or another attribute. Do not tell the participants what has been selected. Call a participant's name and have him/her stand up. Say, "You're a Tibby" if the participant has on the color of the shirt you're thinking of (or other attribute); otherwise, say, "You're not a Tibby". Continue choosing participants that are Tibbys and not Tibbys. Have participants try to guess what makes a participant a Tibby or not a Tibby.

<u>Variations:</u>

- Let participants take the lead and select a characteristic and determine who is and who is not a Tibby.
- Involve two or more attributes in the determination of Tibbys.

Activity: **What's In the Box?**

Format: Whole Group

Objective: Participants will use logical reasoning to determine the number of pieces in the whole set of attribute pieces after asking questions and receiving information about a few items in the set.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces. Before beginning these activities, check your set to be sure all 32 pieces are available: color (red, yellow, blue, green), shape (circle, triangle, square, rhombus or diamond), and size (big, little).

Time Required: 12 minutes

Directions:

1. Tell the participants, "This box (or bag) contains some attribute pieces which have similarities and differences." Shake it so the participants can hear. "I'd like you to ask me some questions with "yes" or "no" answers to figure out what is in the box." If the answer to the question is yes, (i.e., Do you have something red in the box?) the instructor pulls out a block from the box that has this attribute and will help the participants determine the number of items in the entire set (i.e., pull-out a red circle, then if asked again for a red item, pull out a red rhombus, to show there are other attributes beside color, etc.)
2. During the questioning process, stop at certain points and ask the participants:
   - How many pieces do you think I have left in the box?
   - How many pieces would the total set of attribute pieces contain? Why?
   - What are the characteristics (attributes) of this set of attribute pieces? (Size: large or small; Color: red, yellow, green or blue; Shape: square, rhombus, triangle and circle)
   - Is it possible to find a pair of attribute pieces that have neither size, color nor shape in common?
   - Is each of the attribute pieces in your complete set unique?
   - How can you determine the number of pieces in the set of attribute pieces? (Answer: multiply the number of each attribute: Size - 2; Color - 4; Shape - 4; so it is 2x4x4=32 pieces.)

Activity: **Play!**

Format: Whole Group; Small Group; Mini Lecture; etc.

Objectives: Participants have an opportunity for free play with the attribute pieces so that they:

- have an opportunity to informally learn about the characteristics of the pieces by actually handling them; and
- may begin to use the vocabulary of color, size, and shape.

Participants will create a design, pattern, or picture using the small attribute pieces and describe how their "creation" is mathematical.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces.

Time Required: 20 minutes

Directions:

1. The instructor will ask participants to get into partner pairs. The instructor asks one partner to create a design, pattern, or picture using the small attribute pieces; the other, using the large pieces.
2. The instructor asks each partner to convince the other partner that your "creation" is "mathematical."

Activity: **Missing Pieces**

Format: Small Group

Objective: Participants have an opportunity to visualize a whole set and divide into a number of subsets based on the attributes in the set.

Related SOL: K.17, 1.20, 2.25, 3.24, 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces

Time Required: 10 minutes

Directions:

1. The instructor will ask participants to get into partner pairs. The instructor tells the participants to spread the attribute pieces on the table. One partner removes a piece while the other partner looks away. The partner who hides his/her face is asked to identify the missing piece without touching any of the pieces on the table. When the missing piece has been identified, partners are asked to exchange roles and repeat this activity
2. The instructor will ask participants to describe the strategies used to organize the pieces.
3. Extension: Think of a “secret rule” to classify the set of attribute pieces into two piles. Use that rule to slowly sort the pieces as your partner observes. At any time, your partner can call “stop” and guess the rule. After the correct rule identification, players reverse roles. Each incorrect guess results in a one-point penalty. The loser is the first player to accumulate 7 points.

Activity: **20 Questions Game**

Format: Whole Group

Objective: Participants play the “20 Questions” game to develop skill in using the strategy of elimination. The objective of the game is to reduce the set of attribute blocks in the fewest number of questions in order to identify the mystery attribute piece. In this game the participants should develop the strategy of reducing the set by half each time they ask a question, as this strategy always provides the answer within 5 guesses (i.e., $32 = 2^5$).

Related SOL: 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces.

Time Required: 10 minutes

Directions:

1. This is a variation of the standard “20 Questions” game. Ask one participant to think of an attribute piece from the set of attribute pieces. The participant tells the name of the attribute piece to the instructor or writes it on a piece of paper without letting the other participant see it. The other participants, in turn, ask “yes/no” questions about the mystery attribute piece. After each question is answered, the participants move to one side those attribute pieces that do not fit the clues already disclosed. A scorekeeper can count the number of questions asked. Participants try to find the mystery block using the fewest questions possible.
2. After a few games, the instructor can ask, “What is the first best question to eliminate the greatest number of blocks?” The participants may suggest, “Is the block four-sided?” This may not be the best first question, especially when the answer is no. Help the participants recognize that a strategy of eliminating the set by half is the quickest way to reduce the set and identify a specific element. In this game, the participants should learn to reduce the set by half each time, as this strategy always provides the answer within 5 guesses (i.e., $32 = 2^5$).

Game: "Twenty Questions"

Best Strategy: Eliminating Half

Question Number "n"	Eliminate	Left
1	$1/2^1$	*1/2*
2	$1/2^2$	*1/4*
3	$1/2^3$	*1/8*
4	$1/2^4$	*1/16*
5	$1/2^{15}$	*1/32*
n	$1/2^n$	$1/2^n$

When using the 32 attribute pieces, reducing the set so that it has 1/32 left, will result in finding the mystery piece. Questions that eliminate half of the set include:

- Does it have four sides?
- Is it large?
- Is it small?
- Is the color one of the colors in the colors of the U.S. Flag?

Activity: **Who Am I? Game**

Format: Individuals or Small Groups

Objective: Participants reinforce their understanding of attributes through a game where they use clues to identify a specific attribute piece. This is an opportunity to integrate simple rhymes into mathematics.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.10

Materials: Use the 32 piece set of attribute pieces and give participants copies of the appropriate sample Clue Cards.

Time Required: 10 minutes

Directions:
1. This game can be played by individuals or by teams. The players are shown a "clue card." The first player to discover the mystery piece is the winner.
2. Extension: Participants can write their own "Who Am I?" Cards

Answers to the Clue Cards:

32 Piece Attribute Set:
1.) Large, red rhombus
2.) Small, red circle
3.) Large, green rhombus
4.) Small green circle
5.) Large, red triangle

60 piece Attribute Set:
1.) Large thick, red rectangle
2.) Small thin, red circle
3.) Large thin, blue rectangle
4.) Small thin, yellow circle
5.) Large thin, red triangle

Sample Clue Cards For The 32 Piece Attribute Set

(1) *Who am I?* I am large I am not yellow. I have four sides. I am not blue or green. I am not a rhombus. Who am I?	(2) *Who am I?* I am not large. I am green or red. I am not four sided. I have no corners. I am not green. Who am I?
(3) *Who am I?* I do not fit in a round hole. I have four corners. I am not red. I am large. I am green. I am not square. Who am I?	(4) *Who am I?* I am lost, help me find myself. When you find me, hold me in your hand. I am small. I am not blue. I am not square. I am green. I will roll off the table. Who am I?
(5) *Who am I?* I am blue or large or square. I am not green. I am small or a triangle. I am red or blue. I am not a circle. I am blue or large. I am not blue. Who am I?	(6) *Who am I?* Write your own.

Sample Clue Cards For The 60 Piece Attribute Set

(1) *Who am I?* I am large and not a square. I am not yellow. I have four sides. I am not blue or thin. I am not a rhombus. Who am I?	(2) *Who am I?* I am not large. I am yellow or red. I am not four sided. I have no corners. I am not yellow or thick. Who am I?
(3) *Who am I?* I do not fit in a round hole. I have four corners. I am not red. I am large. I am blue and thin. I am not a square. I am not a rhombus. Who am I?	(4) *Who am I?* I am lost, help me find myself. When you find me, hold me in your hand. I am small. I am not blue. I am not square or thick. I am yellow. I will roll off the table. Who am I?
(5) *Who am I?* I am blue or large or square. I am not yellow. I am small or a triangle. I am red or blue. I am not a circle. I am blue or large. I am not blue. Who am I?	(6) *Who am I?* Write your own.

Activity: **Differences - Trains and Games**

Format: Small Group

Objective: Participants reinforce their understanding of differences and similarities through some problem solving activities and games. Participants will identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) as they create difference trains and play games where they must identify the number of differences.

Related SOL: K.17, K.18, 1.20, 1.21, 2.25, 3.24, 4.21, 5.20

Materials: Attribute pieces and the Differences Activity Sheet

Time Required: 20 minutes

Directions:

1. Ask participants to compare attribute pieces in terms of their differences and similarities. Hold up an attribute piece and ask the participants to hold up an attribute piece, which differs in one way. Repeat this with several attribute pieces, and then ask them to hold up an attribute piece, which differs in two ways, then in three ways. At the same time, ask the participants to hold up attribute pieces that are similar in two, one or no ways.
2. Tell the participants that "Difference Trains" have engines and cars. Place the large, red circle on the table as the engine of the train. Cars are to be sequentially attached to the train according to the given rule. Start with the rule that the car to be attached must differ from the preceding car by a single attribute - by one difference. That is, if the engine is a large, red circle, then there are a variety of possibilities that could be attached as the cars; a small, red circle; or a large yellow circle. Have participants identify all of the possibilities. Ask the participants "Why could the small, blue square not be the first car attached to the large, red circle engine? " Taking turns with their partners, have the participants build a train at least 20 cars long, verbalizing the difference as the next car is but into place.
3. Ask the participants "Could you build a train using all of the attribute pieces? Try it!
4. Two-Difference Variation: Have the participants start with the same engine. This time attach a car that differs from the car to which it is attached by two-differences. Ask the participants "Could you build a train using the 16 large pieces before using any small pieces? Try it!"
5. Three-Difference Variation: Have the participants agree on the attribute piece to be the engine. Build a train so that the adjacent cars will differ by exactly three differences.

6. “Differences” is a game for two players or two teams on a four-by-four game mat. Randomly divide the attribute pieces between the two players, so each has 16 pieces.
7. **Game One:** Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its horizontal and vertical neighbors in exactly two ways. The first player who cannot place a block loses.
8. **Game Two:** Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its neighbors horizontally, vertically, and diagonally. The first player who cannot place a block loses.

Differences

Randomly divide the attribute pieces between the two players, so each has 16 pieces.

Game One: Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its horizontal and vertical neighbors in exactly two ways. The first player who cannot place a block loses.

Game Two: Alternate, taking turns to place an attribute piece on the game mat. THE ONLY RULE is that an attribute piece must differ from its neighbors horizontally, vertically, and diagonally. The first player who cannot place a block loses.

Activity: **Hidden Number Patterns**

Format: Individual

Objective: Participants apply their understanding of differences and similarities by identifying number patterns related to the differences in a sequence of attribute pieces.

Related SOL: 4.21, 5.20

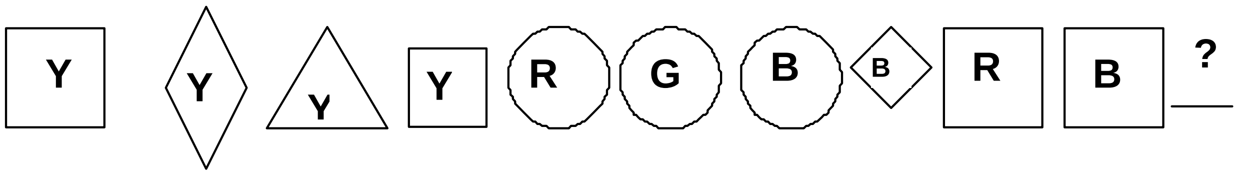
Materials: Use the 32-piece set of attribute pieces and the Hidden Number Activity Sheets.

Time Required: 10 minutes

Directions:

1. Have the participants identify the number of differences between two objects (i.e., one difference, two-differences, three-differences, etc.) in the Hidden Number Activity Sheets (see attachments).
2. Have participants create difference trains where they develop a number pattern for the number of differences in their trains.

HIDDEN NUMBER PATTERN #1



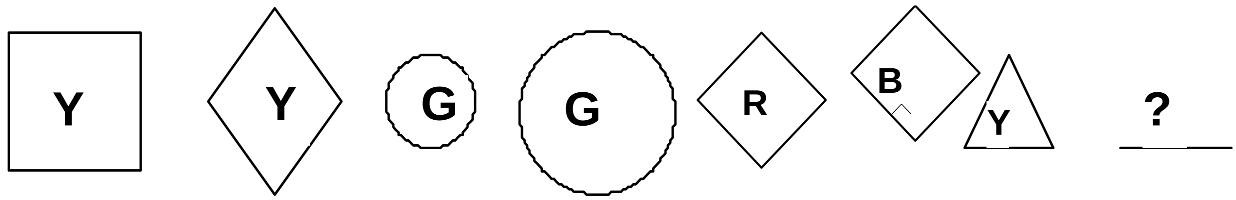
WHAT BLOCK WOULD YOU PLACE AFTER THE LARGE BLUE SQUARE?

WHAT IS THE NUMBER PATTERN (OF DIFFERENCES) ASSOCIATED WITH THIS TRAIN?

ARE THERE ANY OTHER BLOCKS THAT COULD IMMEDIATELY FOLLOW THE LARGE BLUE SQUARE?

WHAT BLOCKS COULD BE PLACED AFTER THE LARGE BLUE SQUARE?

HIDDEN NUMBER PATTERN #2:



WHAT BLOCK WOULD YOU PLACE AFTER THE LARGE YELLOW TRIANGLE?

WHY DID YOU SELECT THAT BLOCK?

ARE THERE ANY OTHER BLOCKS THAT COULD IMMEDIATELY FOLLOW THE LARGE YELLOW TRIANGLE?

WHAT NUMBER PATTERN (OF DIFFERENCES) DID YOU DISCOVER IN THIS TRAIN?

Activity: Attribute Networks

Format: Whole Group, Small Group; Mini Lecture; Assessment

Objective: Participants reinforce their understanding of the relationships by focusing on differences and similarities in an attribute network problem. This may be used as an assessment of a participant's understanding of differences.

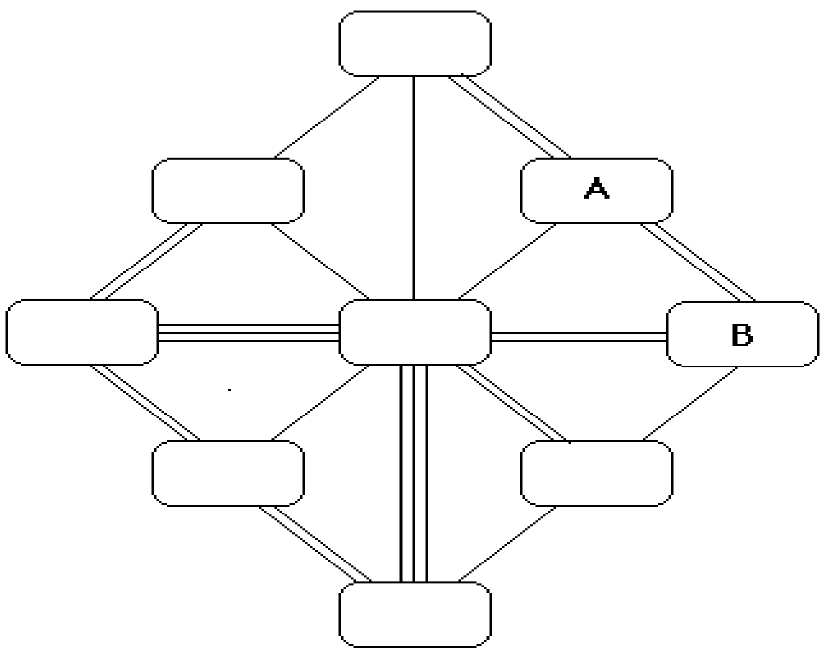
Related SOL: 4.21

Materials: Attribute pieces and a copy of the Attribute Networks Activity Sheet

Time Required: 20 minutes

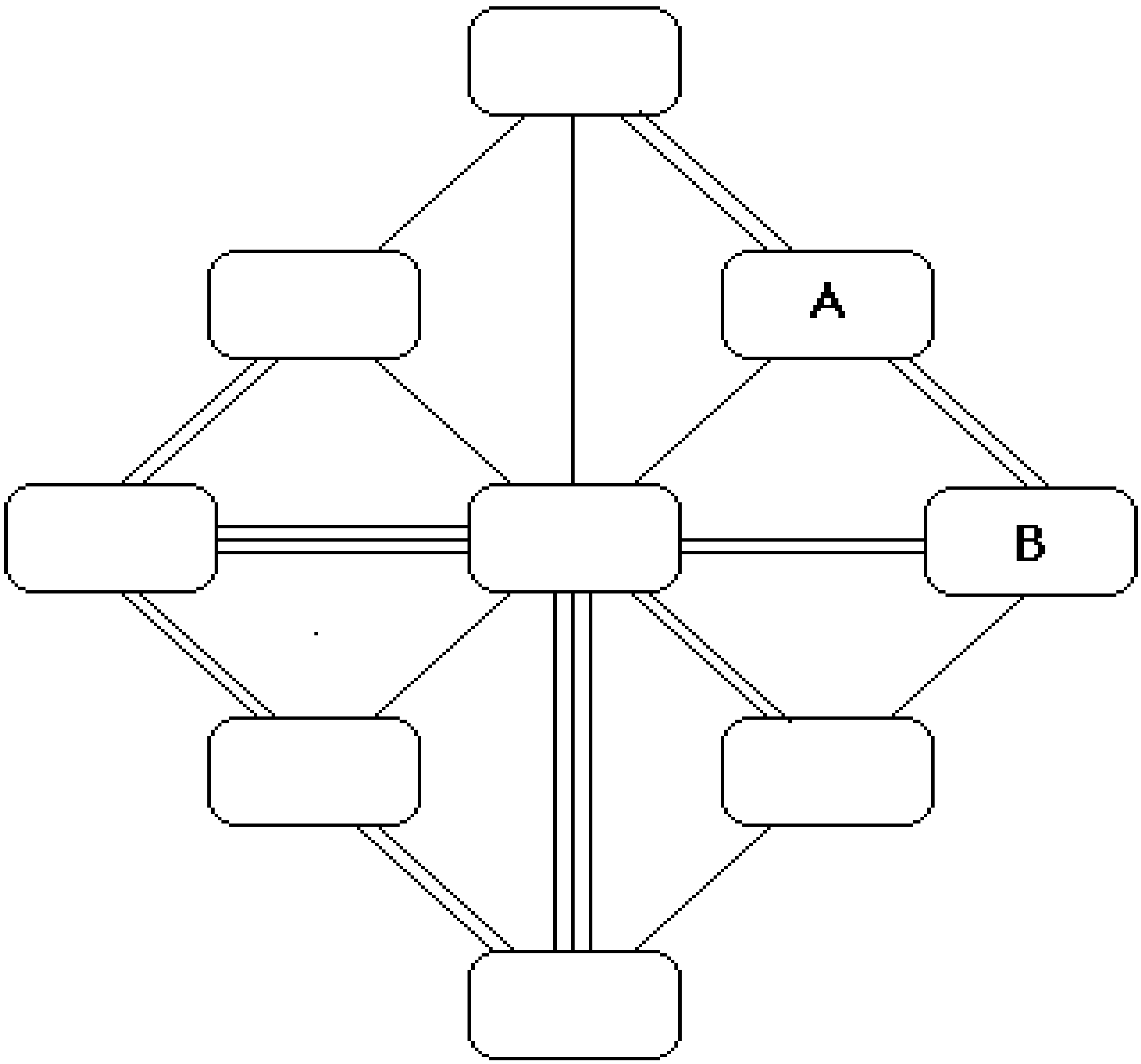
Directions:

1. Place an attribute piece on one of the regions. Place another attribute piece in an adjacent region, which differs from the first piece by as many variables (i.e., color, shape, size) as there are lines connecting the regions. For example, the piece in Region B must differ from the piece in Region A by exactly two attributes.
2. Once you have created a solution that works, write your answers in each block.



NOTE: From A to B, identify blocks that have 2-Differences since there are two lines.

Attribute Networks **Name:**______________________________



Activity: Two-Loop Problems

Format: Whole Group, Small Group; Mini Lecture

Objective: Through problem solving and games, these activities will strengthen the participant's understanding of differences and similarities, which are used in patterning, analyzing and interpreting graphs.

Related SOL: 4.21, 5.20

Materials: Use the 32-piece set of attribute pieces. Before beginning these activities, check your set to be sure all 32 pieces are available. Also have available two pieces of string, each tied in a circle to make two large loops, and a set of Attribute Cards listing one attribute of the set you are using on each card (i.e., red, yellow, square, small, etc.).

Time Required: 30 minutes

Directions:

1. Place the two loops on a table, independent of each other. Select two Attribute Cards, which describe sets with no common pieces. (For example: Squares and Triangles.) Have the participants place the pieces on the table where they belong (either in one of the loops if they are squares or triangles or outside the loops if they are not). After all of the pieces have been placed, have the participants carefully check to see that all the pieces have been correctly placed. Play again with cards such as blue/green; circle/rhombus; etc.
2. Place two overlapping loops on the table. Shuffle the Attribute Cards and place an Attribute Card on each of the loops. Define the sections of the loops. The blocks that would be placed in both loops are placed in the overlapping part of the loops because they have both attributes. For example, if the attribute cards red and triangle were used for the two circles, the intersection would contain only the large red triangle and the small red triangle. Have the participants place the attribute pieces in the loops in their correct positions. Repeat this activity with other Attribute Cards in the loops.
3. Note: The blocks that are in either one loop, the other, and in the overlapping area are in either the red set or the triangle set, or in both. The blocks which are not in the intersection may be described as "triangle but not red," or "red but not triangle."
4. Play the game "In the Loops." This game is played by two players or two teams. Shuffle the Attribute Cards and place them face down in the center of the table. Place the two loops so they overlap. One participant picks two Label Cards and looks at them without showing the other player, then places them face down, one on each loop. The second player chooses a block and asks which section of the loops it belongs or whether it belongs outside the loops. The goal of the game is to name the sets with the fewest number of pieces attempted.

5. As soon as there are sufficient “clues”, the second player tries to name the two sets. When both sets have been named, count the number of pieces the player placed in order to name a set (those both inside and outside the loops). This sum is the player’s score. The winner is the player, or team, with the lowest score after an agreed upon number of turns.
6. Have the participants set out three overlapping loops on the table. Shuffle the Attribute Cards and place an Attribute Card on each of the loops. Define the sections of the loops. The blocks that would be placed in all three loops are placed in the overlapping part of the three loops because they have both attributes. Other overlapping areas of the two loops have the attributes of the two loops. Have the participants try to place all the attribute pieces on the table within the three loops or outside the loops to further reinforce their understanding of intersections.
7. Extension: Play the game “In the Loops” with three overlapping loops.

Attribute Cards:
Print this page on card stock and then cut the pieces out to use for the loop games.

RED	NOT RED
YELLOW	NOT YELLOW
GREEN	NOT GREEN
BLUE	NOT BLUE
LARGE	NOT LARGE
SMALL	NOT SMALL
TRIANGLE	NOT TRIANGLE
SQUARE	NOT SQUARE
RHOMBUS	NOT RHOMBUS
CIRCLE	NOT CIRCLE